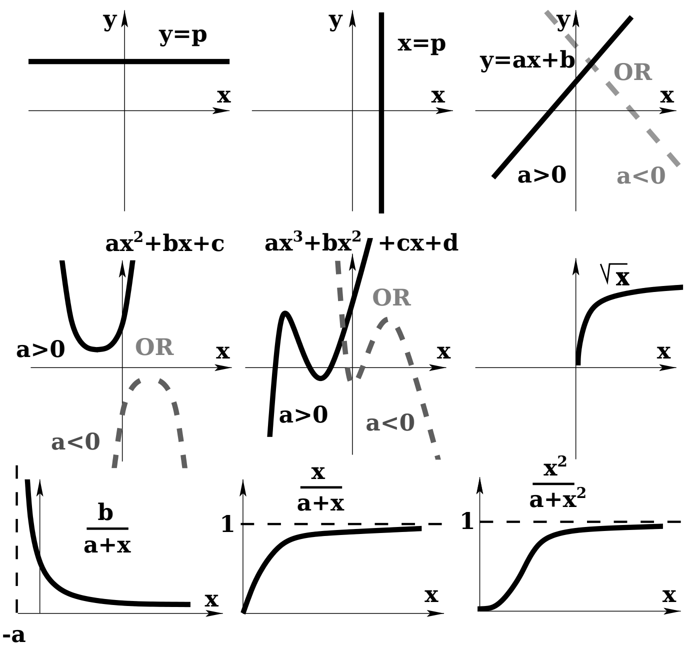
Important graphs:



1D differential equations:
Equation $\frac{\mathrm{d}N}{\mathrm{d}t} = kN$ has the solution: $N(t) = N_0 \mathrm{e}^{kt}$;
N_0 is an (arbitrary) initial value of N. Characteristic time of change is $\tau = 1/k$.

Analysis of 2D systems:

Consider a **general 2D system** $\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = f(x,y) \\ \frac{\mathrm{d}y}{\mathrm{d}t} = g(x,y) \end{cases}$

Equilibria are given by $\begin{cases} f(x,y) = 0 \\ g(x,y) = 0 \end{cases}$

To find the **horizontal x-component of the vectorfield** in a point (x_0, y_0) determine $\frac{\mathrm{d}x}{\mathrm{d}t} = f(x_0, y_0)$,
for $\frac{\mathrm{d}x}{\mathrm{d}t} > 0$ draw $\rightarrow$ for $\frac{\mathrm{d}x}{\mathrm{d}t} < 0$ draw $\leftarrow$
To find the **vertical y-component of the vectorfield** in a point (x_0, y_0) determine $\frac{\mathrm{d}y}{\mathrm{d}t} = g(x_0, y_0)$,
for $\frac{\mathrm{d}y}{\mathrm{d}t} > 0$ draw $\uparrow$ for $\frac{\mathrm{d}y}{\mathrm{d}t} < 0$ draw $\downarrow$

The x**-null-cline** is given by $f(x,y) = 0$, the vector field there is vertical (parallel to the y axis);
The y**-null-cline** is given by $g(x,y) = 0$, the vector field there is horizontal (parallel to the x axis).

To try to determine the **type and stability of an equilibrium** look at the local vectorfield in the four regions of phase space directly surrounding the equilibrium:

Stable node: the vectorfield in all four regions point towards the equilibrium point.
Unstable node: the vectorfield in all four regions point away from the equiibrium point.
Unstable saddle node: the vectorfield points towards the equilibrium in 2 opposite regions of the vectorfield (stable direction) and points away from the equilibrium in the 2 other opposite regions of the vectorfield (unstable direction).

If the vectorfield shows **rotating dynamics** we can have:

- a stable or unstable spiral
- a stable or unstable node
- a center point

and it becomes hard to determine stability and type of the equilibrium

To still try to determine the stability of the equilibrium, look at the **feedback of the variables onto themselves**:

For x, look at a point just right of the equilibrium, and determine the horizontal component of the vectorfield. If it is $\rightarrow$ x has a positive feedback on itself and any increase in x will be amplified, causing instability. If it is $\leftarrow$ x has a negative feedback on itself and any increase in x will be reduced back to the equilibrium, causing stability

For y, look at a point just above the equilibrium, and determine the vertical component of the vectorfield. If it is $\uparrow$ y has a positive feedback on itself and any increase in y will be amplified, causing instability. If it is $\downarrow$ y has a negative feedback on itself and any increase in y will be reduced back to the equilibrium, causing stability

Only if both variables have negative feedback on themselves, or one has negative and one has zero feedback on itself, the equilibrium is stable.

Common equations

equation	solution	conditions
$ax^2+bx+c=0$	$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$ with $D=b^2-4ac$	$a\neq 0$
$x^n=p$	$x=p^{\frac{1}{n}}=\sqrt[n]{p}$	$x>0,\ p>0$
$g^x=c$	$x=\log_g c$	$x>0,\ g>0,\ g\neq 1$
$\log_g x=b$	$x=g^b$	$g>0,\ g\neq 1$
$e^x=c$	$x=\ln c$	$c>0$
$\ln x=b$	$x=e^b$	

Working with powers

Common powers:

$a^0=1$

$a^1=a$

$0^p=0$

$a^{-1}=\frac{1}{a}$

$a^{-p}=\frac{1}{a^p}$

$a^{\frac{1}{2}}=\sqrt{a}$

$a^{\frac{1}{q}}=\sqrt[q]{a}$

$a^{\frac{p}{q}}=(\sqrt[q]{a})^p$

$a^{-\frac{p}{q}}=\frac{1}{(\sqrt[q]{a})^p}$

Rules:

$a^p\times a^q=a^{p+q}$

$\frac{a^p}{a^q}=a^p\times a^{-q}=a^{p-q}$

$\frac{a^p}{b^q}=a^p\times b^{-q}$

$(a^p)^q=a^{pq}$

$(a\times b)^p=a^p\times b^p$

$(a^p\times b^q)^r=(a^p)^r\times(b^q)^r=a^{pr}\times b^{qr}$

Working with fractions

$\frac{a}{b}=\frac{ca}{cb}$

$\frac{a}{b}\times c=\frac{ca}{b}$

$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$

$\frac{ac}{bd}=a\times c\times\frac{1}{b}\times\frac{1}{d}=\frac{a}{b}\times\frac{c}{d}$

$\frac{a}{\frac{b}{c}}=a\times\frac{c}{b}=\frac{ac}{b}$

$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

Logarithms

For a logarithm, the following applies: If $x=n^b$, then $\log_n x=b$, with $n>0$ and $n\neq 1$. For instance, $\log_{10}x$ tells you to what power you should raise 10 (so how many times you should multiply 10 with itself) to get the number x.

The following rules apply to working with logarithms, provided $a,b,n,q>0$ and $n,q\neq 1$:

$\log=\log_{10}$

$\ln=\log_e$

$\log_n ab=\log_n a+\log_n b$

$\log_n\frac{a}{b}=\log_n a-\log_n b$

$\log_n a^p=p\times\log_n a$

$\log_n a=\frac{\log_q a}{\log_q n}$

Derivatives

	function	derivative
	$g(x)=c\times f(x)$	$g'(x)=c\times f'(x)$
sum rule	$p(x)=f(x)+g(x)$	$p'(x)=f'(x)+g'(x)$
product rule	$q(x)=f(x)\times g(x)$	$q'(x)=f'(x)\times g(x)+f(x)\times g'(x)$.
chain rule	$r(x)=f(g(x))$	$r'(x)=f'(g(x))\times g'(x)$ (or $\frac{\mathrm{d}r}{\mathrm{d}x}=\frac{\mathrm{d}r}{\mathrm{d}g}\times\frac{\mathrm{d}g}{\mathrm{d}x}$)
quotient rule	$q(x)=\frac{f(x)}{g(x)}$	$q'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$

Derivatives for some common functions:

$x^n\rightarrow nx^{n-1}$

$e^x\rightarrow e^x$

$g^x\rightarrow g^x\ln g$

$\ln x\rightarrow\frac{1}{x}$

$\log_n x\rightarrow\frac{1}{x\ln n}$

$\sin x\rightarrow\cos x$

$\cos x\rightarrow-\sin x$

$\tan x\rightarrow\frac{1}{\cos^2 x}=1+\tan^2 x$