Answers for bonusquestions Theoretical Biology

For assistants only!

0.1 Chapter 2, question 1, Heck cattle

a The population will be at steady state when the *per capita* birth rate equals the death rate, which happens at a density of about 330 animals (intersection point of the birth and death lines).

b The maximum birth rate $b = 0.4$ per year and the minimum death rate is 0.03 per year. Since we know that at 400 animals the birthrate is approximately 0.2 we can solve $0.4(1-400/k_1) \simeq 0.2$ and estimate that $k_1 = 800$ animals. Since we know that at 500 animals the death rate is approximately 0.33 we can solve $0.03(1+500/k_2) \simeq 0.33$ we can estimate that $k_2 = 50$ animals. When $N = k_1$ the birth rate is zero and when $N = k_2$ the death rate has doubled.

c The full model becomes

$$\frac{\mathrm{d}N}{\mathrm{d}t} = [\beta(N) - \delta(N)]N = [0.4(1-N/800) - 0.03(1+N/50)]N \ .$$

Setting $\mathrm{d}N/\mathrm{d}t = 0$ gives $\bar{N} = 0$ or $0.4(1-N/800) - 0.03(1+N/50) = 0$. This can be rewritten as $0.4 - (0.4N/800) - 0.03 + (0.03N/50) = 0$ and then $0.4 - 0.03 = (0.4/800 + 0.03/50)N$ which finally gives us $\bar{N} = \frac{0.4-0.03}{0.4/800+0.03/50} = 336$ animals.

d The death rate is $\delta(N) = 0.03(1+336/50) = 0.23$ per year (23% per year), which amounts to $0.23 \times 336 = 78$ starving animals per year.

e The $R_0 = b/d = 0.4/0.03 \simeq 13$ offspring per generation. This is normal logistic growth model because there is a positive linear growth term initially causing exponential growth and a negative quadratic term causing saturation of the growth to a steady state population level.

f The number of animals dying from starvation is lowest when about $s = 30$ animals are shot per year, but the total number of animals dying at this hunting rate is 60 animals per year (so still 30 from starvation), which is not that much lower than the 78 animals dying per year in the absence of hunting. Furthermore overall population level has decreased from 336 to 200 animals and as a consequence birth rates are also lower.

g For $s = 31$ the parabola has been shifted downward so much that it touches the x axis, resulting in an equilibrium with one stable and one unstable direction. For $s > 31$ the parabola lies entirely below the x axis, and the population will go extinct.In the abc-formula this is reflected by the term under the square root becoming negative, i.e. there are no longer any equilibrium solutions.

h Starting below the unstable equilibrium leads to extinction of the population.

i GRIND simulations closely mimic the time courses shown in the article.